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不同阶次的分数阶复值混沌系统的广义投影同步和广义错位投影同步

王志成,王　震
(山东科技大学 数学与系统科学学院,山东 青岛 266590)

摘　要: 研究了分数阶复值混沌系统的同步问题。应用不等阶次分数阶实值混沌系统的同步和复值混沌系统的同步方法,提出了广义投影同步和广义错位投影同步。针对驱动系统和响应系统阶次不相同的情况,基于分数阶非线性系统稳定性理论,以复值分数阶 Chen 系统为例,运用自适应控制方法设计反馈控制器,将不等阶分数阶复值系统同步问题转化为可以讨论的等阶复值系统同步问题,并通过理论分析和数值仿真验证了该理论的有效性。

关键词: 分数阶;复值;混沌;同步

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Generalized projective and dislocated projective synchronization of fractional-order complex-valued chaotic systems with different orders

WANG Zhicheng, WANG Zhen
(College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China)

Abstract: In this paper study on synchronization of fractional-order complex-valued chaotic systems is presented. The generalized projective as well as dislocation projective synchronization using the synchronization theory of different fractional-order real-valued chaotic systems and complex-valued chaotic systems is proposed. Considering the different orders of the drive and response systems, and based on stability theory of fractional order nonlinear systems, the complex-valued fractional Chen system as an example is taken. The objective is to design the feedback controller by using the self-adaptive control method and transform the synchronization problem of unequal-order fractional complex-valued system into the achievable synchronization problem of same order complex-valued systems. Theoretical analysis and numerical simulation experiments were then carried out to verify the validity of the theory.

Key words: fractional order; complex-valued; chaos; synchronization

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作者简介: 王志成(1993—),男,山东烟台人,硕士研究生,主要从事分数阶系统控制与同步研究.
王　震(1977—),男,山东泰安人,副教授,博士生导师,主要从事非线性系统理论、神经网络理论、分数阶系统理论及应用研究,本文通信作者. E-mail:wangzhen_sd@126.com

分数阶微积分具有和整数阶微分理论近乎同样长的历史,但由于人们的认知水平不足、缺乏对应的物理应用背景等原因,分数阶微分一直没得到相应的发展和重视[1]。直到 1982 年,Mandelbrot 等[2]第一次指出自然界和许多其他领域中存在很多相似于整数阶系统的分数维现象;在生物医学、力学物理、金融工程和神经网络工程等一些新兴领域,用整数微分方程建模存在很大的局限性,但利用分数阶微积分可以有效改善遗传记忆问题[3-5]。此外,由于混沌信号具有初值敏感性、类随机性、连续宽带谱等特性,分数阶混沌系统在保密通信中具有巨大的潜在价值,可实现数字混沌加密通信,有利于提高信息的安全传输[6-7],因此研究分数阶系统具有十分重要的意义。

早在 1990 年,Peora 和 Corrol[8]就提出了混沌同步的概念,并广泛应用于物理学、气象学等各种工程和物理领域中。近年来,混沌同步在保密通信等跨学科领域的潜在应用价值吸引了许多学者的注意[9],并取得了一些重大成果。例如:自适应、脉冲和滑模变结构等同步方法[10-11],完全同步、反同步、投影同步、函数投影同步等[12-14]。相对于整数阶系统,分数阶系统可以更加准确地描述系统的动态变化,并且控制自由度更高[15],吸引了很多学者对分数阶混沌动力学系统同步进行研究,并取得了一系列进展[16-17]。然而,在工程实践中复值变量更为常见,复值系统在电磁场等领域中具有更重要的应用前景,研究者开展复值系统动力行为的研究,包括分岔、同步和稳定性分析等[17-18]。但目前对于复值分数阶系统和阶次不等的分数阶系统的同步研究还很少[19]。本研究利用混沌同步能增强保密通信的抗破解能力的特点,结合不同阶次分数阶广义投影同步和采用复杂多变的比例因子,提出不同阶次复值分数阶广义错位投影同步,提高保密通信的安全传输。

1 知识准备与问题描述

1.1 分数阶微积分定义

在不同领域的研究中,分数阶微积分是对整数阶微积分的推广,在研究过程的实际应用中,定义主要有 Grunwald-Letnikov 定义、Riemann-Liouville 定义和 Caputo 定义。

Caputo 定义数学表达式如下:

$$ {}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \times \int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\mathrm{d}\tau, \tag{1}$$

其中 $n-1<\alpha<n$,$f(t)$ 为在 $[0,t]$ 上 $n+1$ 阶连续有界可导函数;$\Gamma(\cdot)$ 是伽马函数:

$$\Gamma(z)=\int_0^\infty \mathrm{e}^{-t} t^{z-1}\mathrm{d}t, \qquad \mathrm{Re}z>0。$$

因为分数阶 Caputo 微分定义对初始值有敏感性,所以更多的应用在工程领域,与分数阶微分对应的是积分:

$$D_0^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \times \int_0^t x^{\alpha-1} f(t-\alpha)\mathrm{d}x。\tag{2}$$

1.2 广义投影同步和广义错位投影同步描述

驱动系统和响应系统数学模型:

$$\frac{\mathrm{d}^\alpha \boldsymbol{x}}{\mathrm{d}\, t^\alpha} = f(\boldsymbol{x}) + AF(\boldsymbol{x})(t\geqslant 0), \tag{3}$$

$$\frac{\mathrm{d}^\alpha \boldsymbol{y}}{\mathrm{d}t^\alpha} = g(\boldsymbol{y}) + Bg(\boldsymbol{y}) + u(t)(t\geqslant 0) 。\tag{4}$$

其中 $\boldsymbol{x}=(x_1, x_2, \cdots x_n)^{\mathrm{T}} \in \boldsymbol{R}^n$,$\boldsymbol{y}=(y_1, y_2, \dots y_n)^{\mathrm{T}} \in \mathbf{R}^n$ 分别表示驱动系统和响应系统的时间状态变量;f,g:$\mathbf{R}^n \to \mathbf{R}^n$,$F$,$G$:$\mathbf{R}^{n\times n} \to \mathbf{R}^{n\times n}$ 表示非线性函数;$\boldsymbol{A}$,$\boldsymbol{B}$ 表示参数向量,$u(t)$ 为要设计的非线性控制器。

定义驱动系统和响应系统同步的误差向量:$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{\Delta x}(t)$,其中 $\boldsymbol{\Delta} = \delta_{ij}$ 是比例因子矩阵,$\boldsymbol{\Delta} \in \mathbf{R}^{n\times n}$ 为非奇异矩阵,该矩阵每行每列只有一个非零元素值,若 $\boldsymbol{\Delta}$ 为对角矩阵,该同步称为广义投影同步,若 $\boldsymbol{\Delta}$ 不是对角矩阵,则此同步为广义错位投影同步,要求驱动系统与响应系统的状态变量不是完全一一对应的,而是按照错位关系成比例的同步。

定义 1 对于驱动系统(3)和响应系统(4),如果存在比例因子矩阵 $\boldsymbol{\Delta}$ 和控制器 $u(t)$ 使得 $\lim\limits_{t\to\infty}\|e(t)\|=0$,则称驱动系统(3)与响应系统(4)实现了广义投影(错位)同步。

引理 1[19]　设 $f(t)\in C_a^{\alpha}([c,d])$,$D_a^{\alpha}f(t)\in C_a^{\beta}([c,d])$,$\alpha,\beta>0$ 且 $m-1<\beta<m,n-1<\alpha<n$,则有:

$$D_a^{\beta}(D_a^{\alpha}f(t))=D_a^{\alpha+\beta}f(t)\ 。\tag{5}$$

引理 2[20]　对于一般的分数阶非线性系统:$\frac{\mathrm{d}^{\alpha}\boldsymbol{M}}{\mathrm{d}t^{\alpha}}=A(\boldsymbol{M})\boldsymbol{M}$,当分数阶阶次 $0<\alpha<1$,如果存在正定矩阵 $\boldsymbol{P}$,使得函数 $h=\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P}\frac{\mathrm{d}^{\alpha}\boldsymbol{M}}{\mathrm{d}t^{\alpha}}\leq 0$ 恒成立,则此分数阶系统变量趋于稳定。

2　分数阶复值 Chen 混沌系统及其不同阶次广义投影同步

2.1　分数阶复值 Chen 混沌系统

分数阶复值 Chen 混沌系统的数学模型如下:

$$\begin{cases}\frac{\mathrm{d}^{\alpha}x_1}{\mathrm{d}t^{\alpha}}=a_1(x_3(t)-x_1(t)),\\ \frac{\mathrm{d}^{\alpha}x_2}{\mathrm{d}t^{\alpha}}=a_1(x_4(t)-x_2(t)),\\ \frac{\mathrm{d}^{\alpha}x_3}{\mathrm{d}t^{\alpha}}=(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t),\\ \frac{\mathrm{d}^{\alpha}x_4}{\mathrm{d}t^{\alpha}}=(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t),\\ \frac{\mathrm{d}^{\alpha}x_5}{\mathrm{d}t^{\alpha}}=-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)。\end{cases}\tag{6}$$

为研究该系统的非线性动力学行为,取系统参数和阶数:$a_1=35$,$a_2=3$,$a_3=28$,$\alpha=0.93$,$\alpha=0.98$,分数阶 Chen 系统处于混沌状态,混沌吸引子见图 1 和图 2。

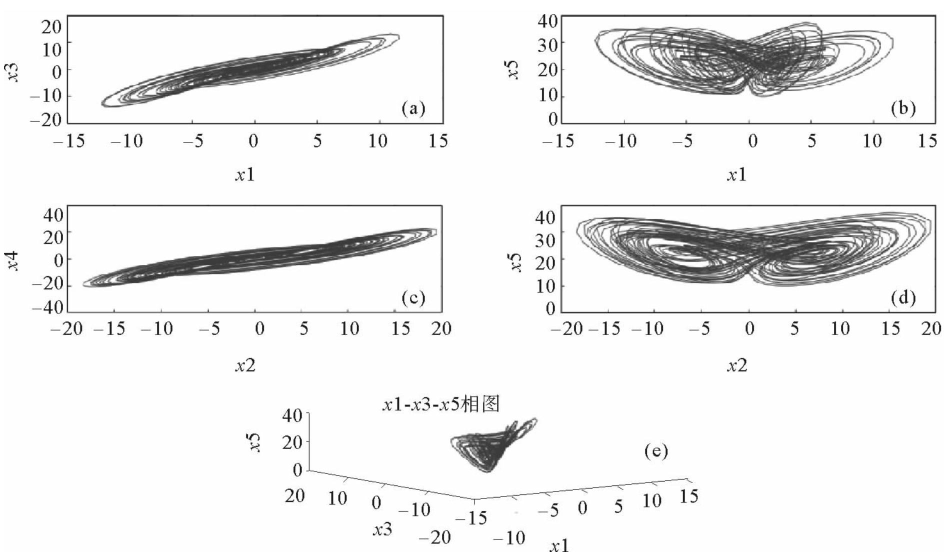

图 1　相空间中的混沌吸引子(α=0.93)

Fig. 1　Chaotic attractor in phase space ($\alpha=0.93$)

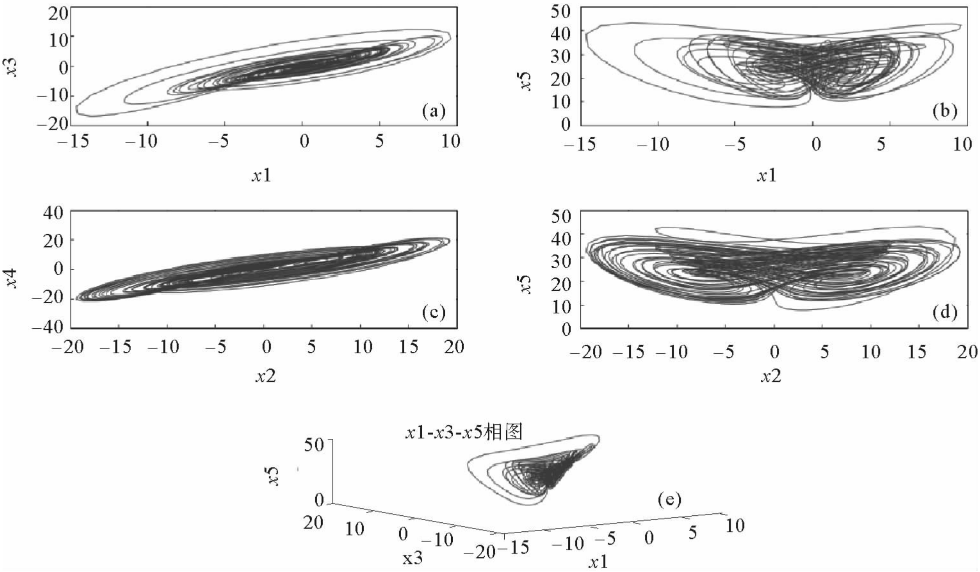

图 2　相空间中的混沌吸引子(α=0.98)

Fig. 2　Chaotic attractor in phase space ($\alpha=0.98$)

2.2　不同阶次分数阶复值 Chen 系统广义投影同步

研究驱动系统和响应系统阶次不等情况下的同步,以系统(6)作为驱动系统,以系统(7)作为响应系统,假设响应系统的阶次β大于驱动系统的阶次$\alpha(0<\alpha<\beta<1)$,有

$$\begin{cases}\dfrac{d^\beta y_1}{dt^\beta}=a_1(y_3(t)-y_1(t))+u_1(t),\\ \dfrac{d^\beta y_2}{dt^\beta}=a_1(y_4(t)-y_2(t))+u_2(t),\\ \dfrac{d^\beta y_3}{dt^\beta}=(a_3-a_1)y_1(t)-y_1(t)y_5(t)+a_3y_3(t)+u_3(t),\\ \dfrac{d^\beta y_4}{dt^\beta}=(a_3-a_1)y_2(t)-y_2(t)y_5(t)+a_3y_4(t)+u_4(t),\\ \dfrac{d^\beta y_5}{dt^\beta}=-a_2y_5(t)+y_1(t)y_3(t)+y_2(t)y_4(t)+u_5(t)。\end{cases}\tag{7}$$

根据分数阶微分的定义和引理 1 知,驱动系统方程(6)可以转化为:

$$\begin{cases}(\dfrac{d^\alpha x_1}{dt^\alpha})^{(\beta-\alpha)}=[a_1(x_3(t)-x_1(t))]^{(\beta-\alpha)},\\ (\dfrac{d^\alpha x_2}{dt^\alpha})^{(\beta-\alpha)}=[a_1(x_4(t)-x_2(t))]^{(\beta-\alpha)},\\ (\dfrac{d^\alpha x_3}{dt^\alpha})^{(\beta-\alpha)}=[(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t)]^{(\beta-\alpha)},\\ (\dfrac{d^\alpha x_4}{dt^\alpha})^{(\beta-\alpha)}=[(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t)]^{(\beta-\alpha)},\\ (\dfrac{d^\alpha x_5}{dt^\alpha})^{(\beta-\alpha)}=[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)]^{(\beta-\alpha)}。\end{cases}\tag{8}$$

即
$$\begin{cases}\frac{d^\beta x_1}{dt^\beta}=[a_1(x_3(t)-x_1(t))]^{(\beta-\alpha)},\\ \frac{d^\beta x_2}{dt^\beta}=[a_1(x_4(t)-x_2(t))]^{(\beta-\alpha)},\\ \frac{d^\beta x_3}{dt^\beta}=[(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t)]^{(\beta-\alpha)},\\ \frac{d^\beta x_4}{dt^\beta}=[(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t)]^{(\beta-\alpha)},\\ \frac{d^\beta x_5}{dt^\beta}=[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)]^{(\beta-\alpha)}。\end{cases}\tag{9}$$

因此不等阶次的分数阶复值混沌 Chen 系统(6)和(7)的同步问题就转化为混沌系统(9)和(7)的同步问题。

定义系统的同步误差：
$$\begin{cases}e_1(t)=y_1(t)-x_1(t),\\ e_2(t)=y_2(t)-x_2(t),\\ e_3(t)=y_3(t)-x_3(t),\\ e_4(t)=y_4(t)-x_4(t),\\ e_5(t)=y_5(t)-x_5(t)。\end{cases}\tag{10}$$

由式(7)、(9)、(10)，得到其同步误差方程：
$$\begin{cases}\frac{d^\beta e_1}{dt^\beta}=a_1(y_3(t)-y_1(t))-[a_1(x_3(t)-x_1(t))]^{(\beta-\alpha)}+u_1(t),\\ \frac{d^\beta e_2}{dt^\beta}=a_1(y_4(t)-y_2(t))-[a_1(x_4(t)-x_2(t))]^{(\beta-\alpha)}+u_2(t),\\ \frac{d^\beta e_3}{dt^\beta}=(a_3-a_1)y_1(t)-y_1(t)y_5(t)+a_3y_3(t)-[(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t)]^{(\beta-\alpha)}+u_3(t),\\ \frac{d^\beta e_4}{dt^\beta}=(a_3-a_1)y_2(t)-y_2(t)y_5(t)+a_3y_4(t)-[(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t)]^{(\beta-\alpha)}+u_4(t),\\ \frac{d^\beta e_5}{dt^\beta}=-a_2y_5(t)+y_1(t)y_3(t)+y_2(t)y_4(t)-[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)]^{(\beta-\alpha)}+u_5(t)。\end{cases}\tag{11}$$

定理 1 基于分数阶稳定性理论和自适应控制方法，设计控制器如下：
$$\begin{cases}u_1(t)=-a_1(x_3(t)-x_1(t))+x_5(t)e_3(t)-a_3e_3(t)-x_3(t)e_5(t)+[a_1(x_3(t)-x_1(t))]^{(\beta-\alpha)},\\ u_2(t)=-a_1(x_4(t)-x_2(t))+x_5(t)e_4(t)-a_3e_4(t)-x_4(t)e_5(t)+[a_1(x_4(t)-x_2(t))]^{(\beta-\alpha)},\\ u_3(t)=-(a_3-a_1)x_1(t)+x_1(t)x_5(t)-a_3x_3(t)-(a_3+1)e_3(t)+[(a_3-a_1)x_1(t)\\ \qquad +a_3x_3(t)-x_1(t)x_5(t)]^{(\beta-\alpha)},\\ u_4(t)=-(a_3-a_1)x_2(t)+x_2(t)x_5(t)-a_3x_4(t)-(a_3+1)e_4(t)+[(a_3-a_1)x_2(t)\\ \qquad +a_3x_4(t)-x_2(t)x_5(t)]^{(\beta-\alpha)},\\ u_5(t)=a_2x_5(t)-x_1(t)x_3(t)-x_2(t)x_4(t)+[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)]^{(\beta-\alpha)}。\end{cases}\tag{12}$$

使误差系统(11)在零点处稳定，即 $\lim_{t\to\infty}\|e(t)\|=0$，则分数阶同步误差系统(11)稳定，即驱动系统(6)和响应系统(7)同步。

证明：把所设计的控制器(12)代入误差系统(11)得到新的同步误差系统：

$$\begin{cases}\dfrac{d^{\beta} e_1}{dt^{\beta}} = a_1(e_3(t)-e_1(t))+x_5(t)e_3(t)-a_3 e_3(t)-x_3(t)e_5(t),\\ \dfrac{d^{\beta} e_2}{dt^{\beta}} = a_1(e_4(t)-e_2(t))+x_5(t)e_4(t)-a_3 e_4(t)-x_4(t)e_5(t),\\ \dfrac{d^{\beta} e_3}{dt^{\beta}} = (a_3-a_1)e_1(t)-y_1(t)y_5(t)+x_1(t)x_5(t)-e_3(t),\\ \dfrac{d^{\beta} e_4}{dt^{\beta}} = (a_3-a_1)e_2(t)-y_2(t)y_5(t)+x_2(t)x_5(t)-e_4(t),\\ \dfrac{d^{\beta} e_5}{dt^{\beta}} = -a_2 e_5(t)+y_1(t)y_3(t)+y_2(t)y_4(t)-x_1(t)x_3(t)-x_2(t)x_4。\end{cases} \tag{13}$$

构造函数 $h_1(e)$:

$$\begin{aligned} h_1(e) = {} & e_1(t)\frac{d^{\beta} e_1}{dt^{\beta}}+e_2(t)\frac{d^{\beta} e_2}{dt^{\beta}}+e_3(t)\frac{d^{\beta} e_3}{dt^{\beta}}+e_4(t)\frac{d^{\beta} e_4}{dt^{\beta}}+e_5(t)\frac{d^{\beta} e_5}{dt^{\beta}}\\ = {} & -a_1e_1^2(t)+a_1e_1(t)e_3(t)+x_5(t)e_1(t)e_3(t)-a_3e_1(t)e_3(t)-x_3(t)e_1(t)e_5(t)\\ & -a_1e_2^2(t)+a_1e_2(t)e_4(t)+x_5(t)e_2(t)e_4(t)-a_3e_2(t)e_4(t)-x_4(t)e_2(t)e_5(t)\\ & +a_3e_1(t)e_3(t)-a_1e_1(t)e_3(t)-x_5(t)e_1(t)e_3(t)-y_1(t)e_3(t)e_5(t)-e_3^2(t)\\ & +a_3e_2(t)e_4(t)-a_1e_2(t)e_4(t)-x_5(t)e_2(t)e_4(t)-y_2(t)e_4(t)e_5(t)-e_4^2(t)\\ & -a_2e_5^2(t)+x_3(t)e_1(t)e_5(t)+y_1(t)e_3(t)e_5(t)+x_4(t)e_2(t)e_5(t)\\ & +y_2(t)e_4(t)e_5(t)\\ = {} & -a_1e_1^2(t)-a_1e_2^2(t)-e_3^2(t)-e_4^2(t)-a_2e_5^2(t)\leqslant 0。\end{aligned} \tag{14}$$

根据引理 2,在控制器(12)的控制下,分数阶误差系统(11)稳定,定理 1 得证。

3 不同阶次分数阶复值 Chen 系统广义错位投影同步

以不同阶次分数阶复值 Chen 系统为例,以(6)为驱动系统,以(7)为响应系统,研究广义错位投影同步,也就是研究系统(9)和(7)的同步问题,因为驱动系统和响应系统阶次都是 5,所以广义错位投影同步有 5!−1=119 种,设 n_i 表示第 i 种错位形式,$i=1,2,3,\cdots,119$,x_j 和 y_j $(j=1,2,3,4,5)$ 表示两个分数阶系统的状态变量,则有如下 119 种组合:

$$\begin{aligned} & n_1:(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_5),(x_5,y_4);\\ & n_2:(x_1,y_1),(x_2,y_2),(x_3,y_4),(x_4,y_5),(x_5,y_3);\\ & n_3:(x_1,y_1),(x_2,y_2),(x_3,y_4),(x_4,y_3),(x_5,y_5);\\ & n_4:(x_1,y_1),(x_2,y_2),(x_3,y_5),(x_4,y_3),(x_5,y_4);\\ & n_5:(x_1,y_1),(x_2,y_2),(x_3,y_5),(x_4,y_4),(x_5,y_3);\\ & \qquad\vdots\\ & n_{119}:(x_1,y_5),(x_2,y_1),(x_3,y_2),(x_4,y_3),(x_5,y_4)。\end{aligned}$$

这里讨论第 119 种组合 n_{119},假设存在非奇异矩阵 $\boldsymbol{\Delta}$,δ_{ij} $(i,j=1,2,3,4,5)$ 是常数:

$$\boldsymbol{\Delta}=\begin{bmatrix}0&\delta_{12}&0&0&0\\0&0&\delta_{23}&0&0\\0&0&0&\delta_{34}&0\\0&0&0&0&\delta_{45}\\\delta_{51}&0&0&0&0\end{bmatrix}。$$

其余形式也可用此法进行类似分析,该形式的广义错位投影同步误差为:

$$\begin{cases} e_1(t)=y_1(t)-\delta_{12}x_2(t),\\ e_2(t)=y_2(t)-\delta_{23}x_3(t),\\ e_3(t)=y_3(t)-\delta_{34}x_4(t),\\ e_4(t)=y_4(t)-\delta_{45}x_5(t),\\ e_5(t)=y_5(t)-\delta_{51}x_1(t)。\end{cases} \tag{15}$$

由(7),(9),(15)式,可得同步误差系统方程:

$$\begin{cases}\dfrac{d^\beta e_1}{dt^\beta}=a_1(y_3(t)-y_1(t))-\delta_{12}\left[a_1(x_4(t)-x_2(t))\right]^{(\beta-\alpha)}+u_1(t),\\ \dfrac{d^\beta e_2}{dt^\beta}=a_1(y_4(t)-y_2(t))-\delta_{23}\left[(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t)\right]^{(\beta-\alpha)}+u_2(t),\\ \dfrac{d^\beta e_3}{dt^\beta}=(a_3-a_1)y_1(t)-y_1(t)y_5(t)+a_3y_3(t)-\delta_{34}\left[(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t)\right]^{(\beta-\alpha)}+u_3(t),\\ \dfrac{d^\beta e_4}{dt^\beta}=(a_3-a_1)y_2(t)-y_2(t)y_5(t)+a_3y_4(t)-\delta_{45}\left[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)\right]^{(\beta-\alpha)}+u_4(t),\\ \dfrac{d^\beta e_5}{dt^\beta}=-a_2y_5(t)+y_1(t)y_3(t)+y_2(t)y_4(t)-\delta_{51}\left[a_1(x_3(t)-x_1(t))\right]^{(\beta-\alpha)}+u_5(t)。\end{cases}\tag{16}$$

定理 2 对于任意给定的非奇异比例因子矩阵 $\boldsymbol{\Delta}$ 和初始值,在自适应控制器(17)的作用下,可实现驱动系统(9)和响应系统(7)的广义错位投影同步,设计的非线性控制器如下:

$$\begin{cases}u_1(t)=-a_1\delta_{34}x_4(t)+a_1\delta_{12}x_2(t)-a_3e_3(t)+\delta_{51}x_1(t)e_3(t)-\delta_{34}x_4(t)e_5(t)\\ \qquad+\delta_{12}\left[a_1(x_4(t)-x_2(t))\right]^{(\beta-\alpha)},\\ u_2(t)=-a_1\delta_{45}x_5(t)+a_1\delta_{23}x_3(t)-a_3e_4(t)+\delta_{51}x_1(t)e_4(t)-\delta_{45}x_5(t)e_5(t)\\ \qquad+\delta_{23}\left[(a_3-a_1)x_1(t)-x_1(t)x_5(t)+a_3x_3(t)\right]^{(\beta-\alpha)},\\ u_3(t)=-a_3\delta_{12}x_2(t)+a_1\delta_{12}x_2(t)-a_3\delta_{34}x_4(t)+\delta_{12}\delta_{51}(t)x_1(t)x_2(t)-(a_3+1)e_3(t)\\ \qquad+\delta_{34}\left[(a_3-a_1)x_2(t)-x_2(t)x_5(t)+a_3x_4(t)\right]^{(\beta-\alpha)},\\ u_4(t)=-a_3\delta_{23}x_3(t)+a_1\delta_{23}x_3(t)-a_3\delta_{45}x_5(t)+\delta_{23}\delta_{51}(t)x_1(t)x_3(t)-(a_3+1)e_4(t)\\ \qquad+\delta_{45}\left[-a_2x_5(t)+x_1(t)x_3(t)+x_2(t)x_4(t)\right]^{(\beta-\alpha)},\\ u_5(t)=a_2\delta_{51}x_1(t)+\delta_{12}\delta_{34}x_2(t)x_4(t)-\delta_{23}\delta_{45}x_3(t)x_5(t)+\delta_{51}\left[a_1(x_3(t)-x_1(t))\right]^{(\beta-\alpha)}。\end{cases}\tag{17}$$

证明:将非线性控制器(17)代入误差系统方程(16)得:

$$\begin{cases}\dfrac{d^\beta e_1}{dt^\beta}=(a_1-a_3)e_3(t)-a_1e_1(t)+\delta_{51}x_1(t)e_3(t)-\delta_{34}x_4(t)e_5(t),\\ \dfrac{d^\beta e_2}{dt^\beta}=(a_1-a_3)e_4(t)-a_1e_2(t)+\delta_{51}x_1(t)e_4(t)-\delta_{45}x_5(t)e_5(t),\\ \dfrac{d^\beta e_3}{dt^\beta}=(a_3-a_1)e_1(t)-e_3(t)-e_1(t)e_5(t)-\delta_{51}x_1(t)e_1(t)-\delta_{12}x_2(t)e_5(t),\\ \dfrac{d^\beta e_4}{dt^\beta}=(a_3-a_1)e_2(t)-e_4(t)-e_2(t)e_5(t)-\delta_{51}x_1(t)e_2(t)-\delta_{23}x_3(t)e_5(t),\\ \dfrac{d^\beta e_5}{dt^\beta}=-a_2e_5(t)+e_1(t)e_3(t)+\delta_{34}x_4(t)e_1(t)+\delta_{12}x_2(t)e_3(t)+e_2(t)e_4(t)\\ \qquad+\delta_{45}x_5(t)e_2(t)+\delta_{23}x_3(t)e_4(t)。\end{cases}\tag{18}$$

构造函数 $h_2(e)$:

$$\begin{aligned}h_2(e)&=e_1(t)\frac{d^\beta e_1}{dt^\beta}+e_2(t)\frac{d^\beta e_2}{dt^\beta}+e_3(t)\frac{d^\beta e_3}{dt^\beta}+e_4(t)\frac{d^\beta e_4}{dt^\beta}+e_5(t)\frac{d^\beta e_5}{dt^\beta}\\ &=(a_1-a_3)e_3(t)e_1(t)-a_1e_1^2(t)+\delta_{51}x_1(t)e_3(t)e_1(t)-\delta_{34}x_4(t)e_5(t)e_1(t)\\ &\quad+(a_1-a_3)e_4(t)e_2(t)-a_1e_2^2(t)+\delta_{51}x_1(t)e_4(t)e_2(t)-\delta_{45}x_5(t)e_5(t)e_2(t)\\ &\quad+(a_3-a_1)e_1(t)e_3(t)-e_3^2(t)-e_1(t)e_5(t)e_3(t)-\delta_{51}x_1(t)e_1(t)e_3(t)-\delta_{12}x_2(t)e_5(t)e_3(t)\\ &\quad+(a_3-a_1)e_2(t)e_4(t)-e_4^2(t)-e_2(t)e_5(t)e_4(t)-\delta_{51}x_1(t)e_2(t)e_4(t)-\delta_{23}x_3(t)e_5(t)e_4(t)\\ &\quad-a_2e_5^2(t)+e_1(t)e_3(t)e_5(t)+\delta_{34}x_4(t)e_1(t)e_5(t)+\delta_{12}x_2(t)e_3(t)e_5(t)+e_2(t)e_4(t)e_5(t)\\ &\quad+\delta_{45}x_5(t)e_2(t)e_5(t)+\delta_{23}x_3(t)e_4(t)e_5(t)\\ &=-a_1e_1^2(t)-a_1e_2^2(t)-e_3^2(t)-e_4^2(t)-a_2e_5^2(t)\leqslant 0。\end{aligned}\tag{19}$$

由于(19)式满足引理 2,误差系统(16)稳定于零点,实现了广义错位投影同步,定理 2 得证。

4 数值仿真

为验证上述方法的有效性,本节采用求解分数阶微分方程计算精度很高的预估校正算法进行数值仿真实验。

对于第 3 部分的数值仿真,初始值取 $x_1 = 1, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 3, y_1 = 0, y_2 = 0, y_3 = -2, y_4 = 0, y_5 = 9$,驱动系统阶次 $\alpha = 0.96$,响应系统阶次 $\beta = 0.98$,其仿真结果如图 3 所示。初始值取 $x_1 = 1, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 3, y_1 = 0, y_2 = -1.5, y_3 = -2, y_4 = 0, y_5 = 7$,驱动系统阶次 $\alpha = 0.93$,响应系统阶次 $\beta = 0.98$,其仿真结果如图 4 所示,图示结果表明在所设计控制器的作用下误差趋于零,可以实现不同阶次复值分数阶混沌系统的同步,验证了所设计控制器的有效性。

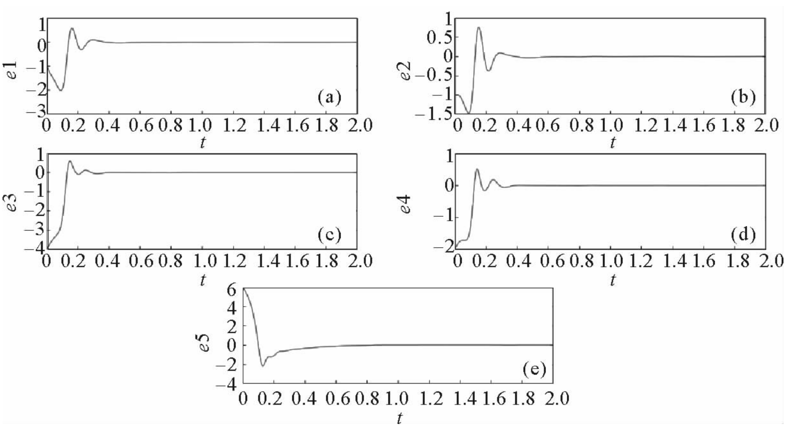

图 3 阶次 0.96 的驱动系统和阶次 0.98 的响应系统的广义投影同步误差

Fig. 3 Generalized projective synchronization error of order 0.96 drive system and 0.98 response system

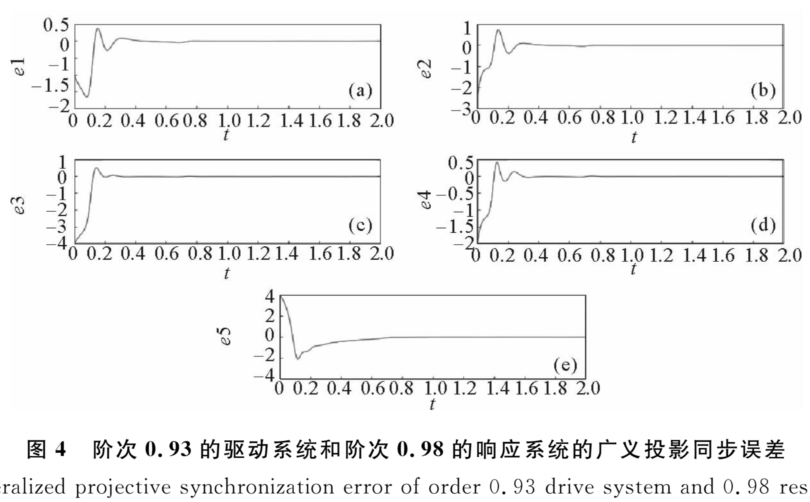

图 4 阶次 0.93 的驱动系统和阶次 0.98 的响应系统的广义投影同步误差

Fig. 4 Generalized projective synchronization error of order 0.93 drive system and 0.98 response system

对于第 4 部分的数值实验,比例因子选取 $\delta_{12} = 3, \delta_{23} = 2.5, \delta_{34} = 2, \delta_{45} = 1.5, \delta_{51} = 3$,驱动响应系统的阶次 $\alpha = 0.96, \beta = 0.98$,初始值选取 $x_1 = 1, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 3, y_1 = 0, y_2 = 0, y_3 = -2, y_4 = 0, y_5 = 9$,同步误差系统图见图 5。驱动响应系统的阶次选取 $\alpha = 0.93, \beta = 0.98$,初始值选取 $x_1 = 1, x_2 = 1, x_3 = 2, x_4 = 2, x_5 = 3, y_1 = 0, y_2 = -1.5, y_3 = -1, y_4 = 0, y_5 = 7$,同步误差系统图见图 6,从图 5 和图 6 可以看出广义错位投影同步误差系统趋于零,驱动系统和响应系统实现了广义错位投影同步,验证了设计的控制器的有效性。

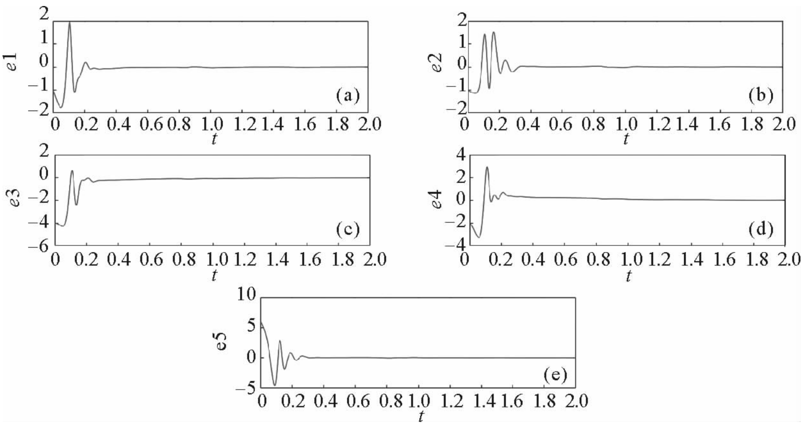

图 5　阶次 0.96 的驱动系统和阶次 0.98 的响应系统的广义错位投影同步误差

Fig. 5　Generalized dislocation projective synchronization error of order 0.96 response system and 0.98 response system

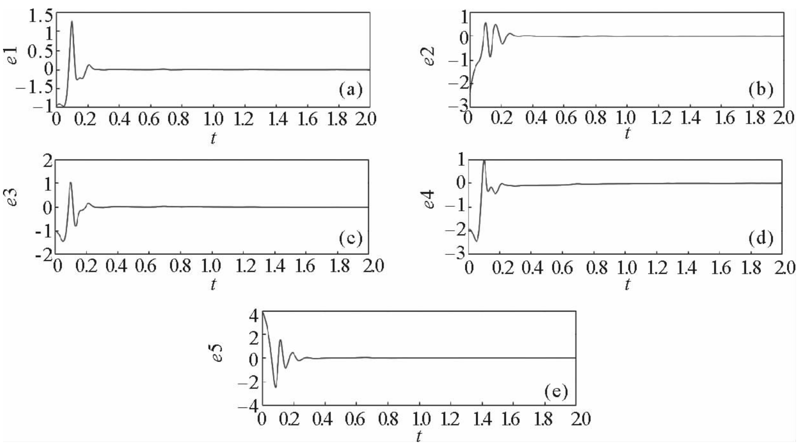

图 6　阶次 0.93 的驱动系统和阶次 0.98 的响应系统的广义错位投影同步误差

Fig. 6　Generalized dislocation projective synchronization error of order 0.93 response system and 0.98 response system

5　结论

依据分数阶微积分的定义和定理,提出一种不同阶次复值分数阶同步的方法。针对不同分数阶阶次的复值混沌系统,可以通过将不同分数阶阶次的复值分数阶系统转化为等阶次的复值分数阶不同结构的系统进行分析,通过设计非线性控制器实现了广义投影同步和广义错位投影同步,并给出了证明。利用预估校正算法进行数值实验,得到的仿真结果与数学理论分析一致,该方法的同步效果与以往的混沌同步效果相同,验证了该方法的正确性。

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