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中立型时滞切换系统的有限时间 有界性和增广耗散性分析

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摘要:研究了中立时滞切换系统有限时间有界及增广耗散性问题,基于平均驻留时间方法和增广耗散性概念,以线性矩阵不等式(LMI)形式,给出使中立时滞切换系统有限时间有界及有限时间增广耗散性的充分条件。最后,数值仿真验证了方法的有效性。

关键词:增广耗散性;有限时间;切换系统;平均驻留时间

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Finite Time Boundedness and Extended Dissipativity Analysis of Switched Neutral Delay Systems

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Abstract: This paper investigates the problem of finite time boundedness and extended dissipativity analysis of switched neutral delay systems. Based on the average dwell-time method and the concept of extended dissipativity, the sufficient conditions for finite time boundedness and extended dissipativity of switched neutral delay systems are derived in the form of linear matrix inequalities (LMIs). The effectiveness of this method is verified by numerical simulations.

Key words: extended dissipativity; finite time; switched system; average dwell-time

切换系统是一类重要的混杂系统,由有限数量的子系统及操纵这些子系统切换的逻辑规则组成,是混杂系统研究的重要方向,不仅有重要的理论价值,而且具有广泛的实际应用背景,如电力系统、机器人控制系统和车辆控制系统等。因此,切换系统得到国内外学者的广泛关注,出现了许多研究成果^[1-5]。众所周知,时滞现象在许多工程系统中广泛存在,使得系统性能变差,甚至是导致系统不稳定的重要原因之一。中立系统是一类特殊的时滞系统,时滞现象不仅存在于状态而且存在于状态导数中。近年来,也涌现出很多关于中立

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系统稳定性^[6-7]、 H_∞ 控制^[8]等方面工作。

值得注意的是,目前大多数关于切换系统性能的文献集中于定义在无穷时间区间上的李雅普诺夫渐近稳定,而在有限时间内的系统动态性能的研究相对较少。文献[9],对一类切换系统引进了有限时间稳定的概念,文献[10]研究了切换系统有限时间有界与 L_2 增益分析等。因此,对于时滞切换系统有限时间性能的进一步研究具有现实意义。同时,Zhang 等^[11]首次提出增广的耗散性概念,即通过矩阵参数调整,使增广的耗散性涵盖一些著名的性能指标如 H_∞ 、 $L_2 - L_\infty$ 、无源性、 (Q, S, R) —耗散性等,更便于系统性能的分析,目前该概念已经被引入神经网络等问题的研究中^[12-13]。

但增广耗散性能指标在切换系统中还少有研究,因此本研究将增广耗散性能指标的研究扩展到中立时滞切换系统,并分析系统的有限时间有界性。首先,给出问题陈述及相关引理,然后基于平均驻留时间以及线性矩阵不等式方法得到中立时滞切换系统有限时间有界且满足有限时间增广的耗散性性能指标的充分条件,最后给出相关结论并通过数值仿真实例验证方法的有效性。

1 问题描述

考虑下面带有时变时滞的中立切换系统:

$$\dot{\mathbf{x}}(t) - C_{\sigma(t)} \dot{\mathbf{x}}(t - \tau(t)) = \mathbf{A}_{\sigma(t)} \mathbf{x}(t) + \mathbf{B}_{\sigma(t)} \mathbf{x}(t - h(t)) + \mathbf{D}_{\sigma(t)} w(t) + \mathbf{G}_{\sigma(t)} \int_{t-r(t)}^t \mathbf{x}(s) ds, \quad (1)$$

$$\mathbf{z}(t) = \mathbf{F}_{\sigma(t)} \mathbf{x}(t), \quad (2)$$

$$\mathbf{x}(\theta) = \boldsymbol{\varphi}(\theta), \forall \theta \in [-\tau, 0]. \quad (3)$$

其中: $\mathbf{x}(t) \in \mathbb{R}^n$ 是状态向量, $w(t) \in \mathbb{R}^n$ 是外部扰动, 属于 $L_2[0, \infty)$, $\mathbf{z}(t) \in \mathbb{R}^n$ 是输出, 切换信号 $\sigma(t): [0, \infty) \rightarrow M = \{1, 2, \dots, i, \dots, l\}$ 是分段连续的, l 是子系统的个数, $\sigma(t) = i$ 表示第 i 个子系统被激活, $\boldsymbol{\varphi}(\theta)$ 是初始状态, $h(t), r(t), \tau(t)$ 是时变时滞并且满足 $0 \leq h(t) \leq h_m, \dot{h}(t) \leq \hat{h} < 1, 0 \leq \tau(t) \leq \tau_m, \tau(t) \leq \hat{\tau} < 1, 0 \leq r(t) \leq r_m$ 。

假设 1 给定常数 T_f , 外部扰动 $w(t)$ 满足 $\int_0^{T_f} \mathbf{w}^T(t) \mathbf{w}(t) dt \leq d, d \geq 0$ 。

假设 2 给定常数 T_f , 状态向量 $\mathbf{x}(t)$ 是时变的并且满足 $\int_0^{T_f} \mathbf{x}^T(t) \mathbf{x}(t) dt \leq k$, 其中 $k \geq 0$ 是一个充分大的常数。

假设 3 矩阵 $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3, \boldsymbol{\psi}_4$ 满足以下条件:

$$1) \boldsymbol{\psi}_1 = \boldsymbol{\psi}_1^T \leq \mathbf{0}, \boldsymbol{\psi}_3 = \boldsymbol{\psi}_3^T \geq \mathbf{0}, \boldsymbol{\psi}_4 = \boldsymbol{\psi}_4^T \geq \mathbf{0};$$

$$2) (\|\boldsymbol{\psi}_1\| + \|\boldsymbol{\psi}_2\|) \|\boldsymbol{\psi}_4\| = \mathbf{0}.$$

假设 4 对 $\forall \alpha \geq 0, \mu \geq 1, \forall t \in [0, T_f]$, 有 $e^{\alpha t} \mu^{N_{\sigma}(0, t)} \leq b$, $N_{\sigma}(0, t)$ 代表 $\sigma(t)$ 在 $(0, t)$ 区间上的切换次数, b 为正的常数。

定义 1^[11] 对给定的矩阵 $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3$ 和 $\boldsymbol{\psi}_4$ 满足假设 3, 当初始状态 $\mathbf{x}(0) = \mathbf{0}$, 系统(1)、(2)具有增广的耗散性,如果对任意的 $T_f \geq 0$ 及 $w(t) \in L_2[0, \infty)$ 以下的不等式成立:

$$\int_0^{T_f} J(t) dt - \sup_{0 \leq t \leq T_f} \mathbf{z}^T(t) \boldsymbol{\psi}_4 \mathbf{z}(t) \geq 0, \quad (4)$$

其中:

$$J(t) = \mathbf{z}^T(t) \boldsymbol{\psi}_1 \mathbf{z}(t) + 2\mathbf{z}^T(t) \boldsymbol{\psi}_2 w(t) + \mathbf{w}^T(t) \boldsymbol{\psi}_3 \mathbf{w}(t).$$

在定义 1 中,通过调整矩阵参数,增广的耗散性涵盖了一些著名的性能指标:

$$1) L_2 - L_\infty \text{ 性能指标: } \boldsymbol{\psi}_1 = \mathbf{0}, \boldsymbol{\psi}_2 = \mathbf{0}, \boldsymbol{\psi}_3 = \gamma^2 \mathbf{I}, \boldsymbol{\psi}_4 = \mathbf{I};$$

$$2) H_\infty \text{ 性能指标: } \boldsymbol{\psi}_1 = -\mathbf{I}, \boldsymbol{\psi}_2 = \mathbf{0}, \boldsymbol{\psi}_3 = \gamma^2 \mathbf{I}, \boldsymbol{\psi}_4 = \mathbf{0};$$

$$3) \text{无源性: } \boldsymbol{\psi}_1 = \mathbf{0}, \boldsymbol{\psi}_2 = \mathbf{I}, \boldsymbol{\psi}_3 = \gamma \mathbf{I}, \boldsymbol{\psi}_4 = \mathbf{0};$$

4) (Q, S, R) - 耗散性: $\psi_1 = Q, \psi_2 = S, \psi_3 = R - \beta I, \psi_4 = \theta$ 。

定义 2 [10] 给定三个常数 c_1, c_2, T_f 且 $c_1 < c_2$, 正定矩阵 R 与切换信号 $\sigma(t)$, 中立切换系统 (1) ~ (3) 基于 $(c_1, c_2, R, T_f, \sigma)$ 是有限时间有界的, 如果对 $\forall t \in [0, T_f]$

$$\sup_{-\tau \leqslant \theta \leqslant 0} \{x^T(\theta)Rx(\theta), \dot{x}^T(\theta)R\dot{x}(\theta)\} \leqslant c_1 \Rightarrow x^T(t)Rx(t) \leqslant c_2。 \quad (5)$$

成立, 若 $w(t) = \theta$, 则称为有限时间稳定的。

定义 3 [10] 对任意 $T_2 > T_1 \geqslant 0$, $N_\sigma(T_1, T_2)$ 代表 $\sigma(t)$ 在时间区间 (T_1, T_2) 上的切换次数, 如果

$$N_\sigma(T_1, T_2) \leqslant N_0 + \frac{T_2 - T_1}{\tau_a}。$$

对于 $\tau_a > 0$ 和整数 $N_0 \geqslant 0$ 都成立, 则 τ_a 称为平均驻留时间。

N_0 为扰动界, 不失一般性, 本文取 $N_0 = 0$ 。

引理 1 [14] X, Y 为适当维数的实向量, 则 $2X^T Y \leqslant X^T X + Y^T Y$ 成立。

引理 2 [14] 对正定矩阵 $N \in \mathbb{R}^{n \times n}$, 标量 $\tau > 0$ 和向量函数 $x(\cdot): \mathbb{R} \rightarrow \mathbb{R}^n$, 积分不等式

$$-\tau \int_{t-\tau}^t x^T(s) Nx(s) ds \leqslant - \int_{t-\tau}^t x^T(s) ds N \int_{t-\tau}^t x(s) ds \text{ 成立。}$$

2 主要结果

2.1 有限时间有界分析

定理 1 考虑系统 (1) ~ (3), 令 $\tilde{P}_i = R^{\frac{1}{2}} P_i R^{\frac{1}{2}}$, $\tilde{Q}_i = R^{\frac{1}{2}} Q_i R^{\frac{1}{2}}$, $\tilde{Z}_i = R^{\frac{1}{2}} Z_i R^{\frac{1}{2}}$, $\tilde{T}_i = R^{\frac{1}{2}} T_i R^{\frac{1}{2}}$, $\tilde{M}_i = R^{\frac{1}{2}} M_i R^{\frac{1}{2}}$ 。对给定正的标量 $\alpha, h_m, \hat{h}, \hat{\tau}, r_m$, 如果存在适当维数的对称正定矩阵 $\tilde{P}_i, \tilde{Q}_i, \tilde{Z}_i, \tilde{T}_i, \tilde{M}_i, N_i$, 以及适当维数的矩阵 $X_{11i}, X_{12i}, X_{22i}, H_{1i}, H_{2i}$ 使得

$$X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geqslant 0, \quad (6)$$

$$\Delta_i = \begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \tilde{T}_i \end{bmatrix} \geqslant 0, \quad (7)$$

$$\Theta_i = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \tilde{P}_i G_i + A_i^T \tilde{Z}_i G_i + h_m A_i^T \tilde{T}_i G_i \\ * & \varphi_{22} & \varphi_{23} & B_i^T \tilde{Z}_i D_i + h_m B_i^T \tilde{T}_i D_i & B_i^T \tilde{Z}_i G_i + h_m B_i^T \tilde{T}_i G_i \\ * & * & \varphi_{33} & C_i^T \tilde{Z}_i D_i + h_m C_i^T \tilde{T}_i D_i & C_i^T \tilde{Z}_i G_i + h_m C_i^T \tilde{T}_i G_i \\ * & * & * & \varphi_{44} & D_i^T \tilde{Z}_i G_i + h_m D_i^T \tilde{T}_i G_i \\ * & * & * & * & G_i^T \tilde{Z}_i G_i + h_m G_i^T \tilde{T}_i G_i - \frac{\tilde{M}_i}{r_m} \end{bmatrix} < 0,$$

其中,

$$\begin{aligned} \varphi_{11} &= -\alpha \tilde{P}_i + \tilde{P}_i A_i + A_i^T \tilde{P}_i + \tilde{Q}_i + A_i^T \tilde{Z}_i A_i + h_m A_i^T \tilde{T}_i A_i + r_m \tilde{M}_i + H_{1i} + H_{2i}^T + h_m X_{11i}, \\ \varphi_{12} &= \tilde{P}_i B_i + A_i^T \tilde{Z}_i B_i + h_m A_i^T \tilde{T}_i B_i - H_{1i}^T + H_{2i} + h_m X_{12i}, \\ \varphi_{13} &= \tilde{P}_i C_i + A_i^T \tilde{Z}_i C_i + h_m A_i^T \tilde{T}_i C_i, \\ \varphi_{14} &= \tilde{P}_i D_i + A_i^T \tilde{Z}_i D_i + h_m A_i^T \tilde{T}_i D_i, \\ \varphi_{22} &= -(1 - \hat{h}) \tilde{Q}_i + B_i^T \tilde{Z}_i B_i + h_m B_i^T \tilde{T}_i B_i - H_{2i}^T - H_{2i} + h_m X_{22i}, \\ \varphi_{23} &= B_i^T \tilde{Z}_i C_i + h_m B_i^T \tilde{T}_i C_i, \\ \varphi_{33} &= -(1 - \hat{\tau}) \tilde{Z}_i + C_i^T \tilde{Z}_i C_i + h_m C_i^T \tilde{T}_i C_i, \\ \varphi_{44} &= -N_i + D_i^T \tilde{Z}_i D_i + h_m D_i^T \tilde{T}_i D_i. \end{aligned} \quad (8)$$

成立, 同时,

$$(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d < c_2 \lambda_1 e^{-\alpha T_f}。 \quad (9)$$

进一步,平均驻留时间满足:

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln[(\lambda_2 + h_m e^{a h_m} \lambda_3 + r_m e^{a r_m} \lambda_4 + h_m e^{a h_m} \lambda_5 + r_m e^{a r_m} \lambda_6) c_1 + \lambda_7 d] - a T_f}, \quad (10)$$

其中

$$\lambda_{\min}(\mathbf{P}_i) = \lambda_1, \lambda_{\max}(\mathbf{P}_i) = \lambda_2, \lambda_{\max}(\mathbf{Q}_i) = \lambda_3, \lambda_{\max}(\mathbf{Z}_i) = \lambda_4, \lambda_{\max}(\mathbf{T}_i) = \lambda_5, \lambda_{\max}(\mathbf{M}_i) = \lambda_6, \lambda_{\max}(\mathbf{N}_i) = \lambda_7. \quad (11)$$

且 $\mu > 1$ 满足

$$\tilde{\mathbf{P}}_i < \mu \tilde{\mathbf{P}}_j, \tilde{\mathbf{Q}}_i < \mu \tilde{\mathbf{Q}}_j, \tilde{\mathbf{Z}}_i < \mu \tilde{\mathbf{Z}}_j, \tilde{\mathbf{T}}_i < \mu \tilde{\mathbf{T}}_j, \tilde{\mathbf{M}}_i < \mu \tilde{\mathbf{M}}_j, \forall i, j \in M. \quad (12)$$

则切换系统 (1) ~ (3) 基于 $(c_1, c_2, T_f, d, R, \sigma)$ 是有限时间有界的。

证明 选取候选李雅普诺夫函数为

$$V(t) = V_{\sigma(t)}(t) = V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t) + V_{5i}(t).$$

$$V_{1i}(t) = \mathbf{x}^T(t) \tilde{\mathbf{P}}_i \mathbf{x}(t),$$

$$V_{2i}(t) = \int_{t-h(t)}^t e^{a(t-s)} \mathbf{x}^T(s) \tilde{\mathbf{Q}}_i \mathbf{x}(s) ds,$$

$$V_{3i}(t) = \int_{t-\tau(t)}^t e^{a(t-s)} \dot{\mathbf{x}}^T(s) \tilde{\mathbf{Z}}_i \dot{\mathbf{x}}(s) ds,$$

$$V_{4i}(t) = \int_{-h_m t + \epsilon}^0 \int_s^t e^{a(t-s)} \dot{\mathbf{x}}^T(s) \tilde{\mathbf{T}}_i \dot{\mathbf{x}}(s) ds d\epsilon,$$

$$V_{5i}(t) = \int_{-r_m t + \epsilon}^0 \int_s^t e^{a(t-s)} \mathbf{x}^T(s) \tilde{\mathbf{M}}_i \mathbf{x}(s) ds d\epsilon.$$

其中: α 是一个标量, $\tilde{\mathbf{P}}_i, \tilde{\mathbf{Q}}_i, \tilde{\mathbf{Z}}_i, \tilde{\mathbf{T}}_i, \tilde{\mathbf{M}}_i$ 均为未知的正定矩阵。

基于系统 (1) ~ (3) 对 $V(t)$ 求导得:

$$\dot{V}_{1i}(t) = 2\mathbf{x}^T(t) \tilde{\mathbf{P}}_i \dot{\mathbf{x}}(t),$$

$$\begin{aligned} \dot{V}_{2i}(t) &= \alpha V_{2i}(t) + \mathbf{x}^T(t) \tilde{\mathbf{Q}}_i \mathbf{x}(t) - e^{a h(t)} (1 - h(t)) \mathbf{x}^T(t - h(t)) \tilde{\mathbf{Q}}_i \mathbf{x}(t - h(t)) \\ &\leqslant \alpha V_{2i}(t) + \mathbf{x}^T(t) \tilde{\mathbf{Q}}_i \mathbf{x}(t) - (1 - \hat{h}) \mathbf{x}^T(t - h(t)) \tilde{\mathbf{Q}}_i \mathbf{x}(t - h(t)), \end{aligned}$$

$$\begin{aligned} \dot{V}_{3i}(t) &= \alpha V_{3i}(t) + \dot{\mathbf{x}}^T(t) \tilde{\mathbf{Z}}_i \dot{\mathbf{x}}(t) - e^{a \tau(t)} (1 - \tau(t)) \dot{\mathbf{x}}^T(t - \tau(t)) \tilde{\mathbf{Z}}_i \dot{\mathbf{x}}(t - \tau(t)) \\ &\leqslant \alpha V_{3i}(t) + \dot{\mathbf{x}}^T(t) \tilde{\mathbf{Z}}_i \dot{\mathbf{x}}(t) - (1 - \hat{\tau}) \dot{\mathbf{x}}^T(t - \tau(t)) \tilde{\mathbf{Z}}_i \dot{\mathbf{x}}(t - \tau(t)), \end{aligned}$$

$$\begin{aligned} \dot{V}_{4i}(t) &= \alpha V_{4i}(t) + h_m \dot{\mathbf{x}}^T(t) \tilde{\mathbf{T}}_i \dot{\mathbf{x}}(t) - \int_{t-h(t)}^t e^{a(t-s)} \dot{\mathbf{x}}^T(s) \tilde{\mathbf{T}}_i \dot{\mathbf{x}}(s) ds \\ &\leqslant \alpha V_{4i}(t) + h_m \dot{\mathbf{x}}^T(t) \tilde{\mathbf{T}}_i \dot{\mathbf{x}}(t) - \int_{t-h(t)}^t \dot{\mathbf{x}}^T(s) \tilde{\mathbf{T}}_i \dot{\mathbf{x}}(s) ds, \end{aligned}$$

$$\begin{aligned} \dot{V}_{5i}(t) &= \alpha V_{5i}(t) + r_m \mathbf{x}^T(t) \tilde{\mathbf{M}}_i \mathbf{x}(t) - \int_{t-r(t)}^t e^{a(t-s)} \mathbf{x}^T(s) \tilde{\mathbf{M}}_i \mathbf{x}(s) ds \\ &\leqslant \alpha V_{5i} + r_m \mathbf{x}^T(t) \tilde{\mathbf{M}}_i \mathbf{x}(t) - \int_{t-r(t)}^t \mathbf{x}^T(s) \tilde{\mathbf{M}}_i \mathbf{x}(s) ds. \end{aligned}$$

由牛顿-莱布尼茨公式得:

$$2[\mathbf{x}^T(t) \mathbf{H}_{1i} + \mathbf{x}^T(t - h(t)) \mathbf{H}_{2i}] \left[\mathbf{x}(t) - \int_{t-h(t)}^t \dot{\mathbf{x}}(s) ds - \mathbf{x}(t - h(t)) \right] = 0,$$

令 $\chi(t) = [\mathbf{x}^T(t) \mathbf{x}^T(t - h(t))]^T$, 则

$$h_m \chi^T(t) \mathbf{X}_i \chi(t) - \int_{t-h(t)}^t \chi^T(s) \mathbf{X}_i \chi(s) ds \geqslant 0$$

显然成立。

又因为

$$\dot{V}(t) - \alpha V(t) - \mathbf{w}^T(t) \mathbf{N}_i \mathbf{w}(t) \leqslant \mathbf{X}^T(t) \mathbf{\Theta}_i \mathbf{X}(t) - \int_{t-h(t)}^t \mathbf{g}^T(t,s) \mathbf{\Lambda}_i \mathbf{g}(t,s) ds,$$

其中,

$$\begin{aligned} \mathbf{X}^T(t) &\stackrel{\Delta}{=} [\mathbf{x}^T(t) \mathbf{x}^T(t-h(t)) \dot{\mathbf{x}}^T(t-\tau(t)) \mathbf{w}^T(t) \int_{t-r(t)}^t \mathbf{x}^T(s) ds], \\ \mathbf{g}^T(t,s) &\stackrel{\Delta}{=} [\mathbf{x}^T(t) \mathbf{x}^T(t-h(t)) \dot{\mathbf{x}}^T(s)], \end{aligned}$$

由引理2得

$$-\int_{t-r(t)}^t \mathbf{x}^T(s) \widetilde{\mathbf{M}}_i \mathbf{x}(s) ds \leqslant -\frac{1}{r_m} \left(\int_{t-r(t)}^t \mathbf{x}(s)^T ds \right)^T \widetilde{\mathbf{M}}_i \left(\int_{t-r(t)}^t \mathbf{x}(s) ds \right).$$

因此,在状态 $\mathbf{x}(t) \neq \mathbf{0}$ 的条件下,由式(7)~(8)得

$$\dot{V}(t) - \alpha V(t) - \mathbf{w}^T(t) \mathbf{N}_i \mathbf{w}(t) < 0. \quad (13)$$

积分式(13),由式(12)~(13)得对 $\forall t \in [t_k, t_{k+1})$,

$$\begin{aligned} V(t) &< e^{a(t-t_k)} V(t_k) + \int_{t_k}^t e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds < e^{a(t-t_k)} \mu V(t_k^-) + \int_{t_k}^t e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds \\ &< e^{a(t-t_k)} \mu \left[e^{a(t_k-t_{k-1})} V(t_{k-1}) + \int_{t_{k-1}}^{t_k} e^{a(t_k-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds \right] + \int_{t_k}^t e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds \\ &= e^{a(t-t_{k-1})} \mu V(t_{k-1}) + \mu \int_{t_{k-1}}^{t_k} e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds + \int_{t_k}^t e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds < \dots \\ &< e^{a(t-0)} \mu^{N_\sigma(0,t)} V(0) + \mu^{N_\sigma(0,t)} \int_0^{t_1} e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds + \mu^{N_\sigma(t_1,t)} \int_{t_1}^{t_2} e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds + \dots \\ &+ \mu \int_{t_{k-1}}^{t_k} e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds + \int_{t_k}^t e^{a(t-s)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds = e^{a(t-0)} \mu^{N_\sigma(0,t)} V(0) \\ &+ \int_0^t e^{a(t-s)} \mu^{N_\sigma(s,t)} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds < e^{at} \mu^{N_\sigma(0,t)} V(0) + \mu^{N_\sigma(0,t)} e^{at} \int_0^t \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds \\ &< e^{aT_f} \mu^{N_\sigma(0,T_f)} \left[V(0) + \int_0^{T_f} \mathbf{w}^T(s) \mathbf{N}_i \mathbf{w}(s) ds \right] < e^{aT_f} \mu^{N_\sigma(0,T_f)} [V(0) + \lambda_{\max}(\mathbf{N}_i)]. \end{aligned} \quad (14)$$

由定义3得:

$$N_\sigma(0, T_f) < \frac{T_f}{\tau_a}. \quad (15)$$

另一方面,

$$\begin{aligned} V(t) &> \mathbf{x}^T(t) \widetilde{\mathbf{P}}_i \mathbf{x}(t) = \mathbf{x}^T(t) \mathbf{R}^{\frac{1}{2}} \mathbf{P}_i \mathbf{R}^{\frac{1}{2}} \mathbf{x}(t) \geqslant \lambda_{\min}(\mathbf{P}_i) \mathbf{x}^T(t) \mathbf{R} \mathbf{x}(t) = \lambda_1 \mathbf{x}^T(t) \mathbf{R} \mathbf{x}(t) \\ V(0) &\leqslant \lambda_{\max}(\mathbf{P}_i) \mathbf{x}^T(0) \mathbf{R} \mathbf{x}(0) + h_m e^{a h_m} \lambda_{\max}(\mathbf{Q}_i) \sup_{-\tau \leqslant \theta \leqslant 0} \{ \mathbf{x}^T(\theta) \mathbf{R} \mathbf{x}(\theta), \dot{\mathbf{x}}^T(\theta) \mathbf{R} \dot{\mathbf{x}}(\theta) \} \\ &+ \tau_m e^{a \tau_m} \lambda_{\max}(\mathbf{Z}_i) \sup_{-\tau \leqslant \theta \leqslant 0} \{ \mathbf{x}^T(\theta) \mathbf{R} \mathbf{x}(\theta), \dot{\mathbf{x}}^T(\theta) \mathbf{R} \dot{\mathbf{x}}(\theta) \} \\ &+ h_m e^{a h_m} \lambda_{\max}(\mathbf{T}_i) \sup_{-\tau \leqslant \theta \leqslant 0} \{ \mathbf{x}^T(\theta) \mathbf{R} \mathbf{x}(\theta), \dot{\mathbf{x}}^T(\theta) \mathbf{R} \dot{\mathbf{x}}(\theta) \} \\ &+ r_m e^{a r_m} \lambda_{\max}(\mathbf{M}_i) \sup_{-\tau \leqslant \theta \leqslant 0} \{ \mathbf{x}^T(\theta) \mathbf{R} \mathbf{x}(\theta), \dot{\mathbf{x}}^T(\theta) \mathbf{R} \dot{\mathbf{x}}(\theta) \} \\ &\leqslant [\lambda_{\max}(\mathbf{P}_i) + h_m e^{a h_m} \lambda_{\max}(\mathbf{Q}_i) + \tau_m e^{a \tau_m} \lambda_{\max}(\mathbf{Z}_i) + h_m e^{a h_m} \lambda_{\max}(\mathbf{T}_i) + r_m e^{a r_m} \lambda_{\max}(\mathbf{M}_i)] \\ &\quad \sup_{-\tau \leqslant \theta \leqslant 0} \{ \mathbf{x}^T(\theta) \mathbf{R} \mathbf{x}(\theta), \dot{\mathbf{x}}^T(\theta) \mathbf{R} \dot{\mathbf{x}}(\theta) \} \end{aligned} \quad (16)$$

$$\leq (\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1. \quad (17)$$

由式(15)~(17)得

$$x^T(t) R x(t) \leq \frac{V(t)}{\lambda_1} < \frac{(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d}{\lambda_1} e^{a T_f} u^{\frac{T_f}{\tau_a}}. \quad (18)$$

当 $\mu = 1$, 由式(9)得

$$x^T(t) R x(t) < c_2.$$

当 $\mu > 1$, 由式(9)得

$$\ln(\lambda_1 c_2) - \ln[(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d] - \alpha T_f > 0$$

由(10)得

$$\begin{aligned} \frac{T_f}{\tau_a} &< \frac{\ln(\lambda_1 c_2) - \ln[(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d] - \alpha T_f}{\ln(\mu)} \\ &= \frac{\ln \left[\frac{\lambda_1 c_2 e^{-\alpha T_f}}{(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d} \right]}{\ln(\mu)} > 0. \end{aligned} \quad (19)$$

把式(19)代入式(18)得

$$\begin{aligned} x^T(t) R x(t) &< \left[\frac{(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d}{\lambda_1} \right] \\ &\quad e^{\alpha T_f} \left[\frac{c_2 \lambda_1 e^{-\alpha T_f}}{(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 + \lambda_7 d} \right] = c_2, \end{aligned}$$

证毕。

推论 考虑系统(1)~(3), 令 $w(t) \equiv 0$, $\tilde{P}_i = \mathbf{R}^{\frac{1}{2}} P_i \mathbf{R}^{\frac{1}{2}}$, $\tilde{Q}_i = \mathbf{R}^{\frac{1}{2}} Q_i \mathbf{R}^{\frac{1}{2}}$, $\tilde{Z}_i = \mathbf{R}^{\frac{1}{2}} Z_i \mathbf{R}^{\frac{1}{2}}$, $\tilde{T}_i = \mathbf{R}^{\frac{1}{2}} T_i \mathbf{R}^{\frac{1}{2}}$, $\tilde{M}_i = \mathbf{R}^{\frac{1}{2}} M_i \mathbf{R}^{\frac{1}{2}}$ 。对给定正的标量 $\alpha, h_m, \hat{h}, \hat{\tau}, r_m$, 存在适当维数的正定对称矩阵 $\tilde{P}_i, \tilde{Q}_i, \tilde{Z}_i, \tilde{T}_i, \tilde{M}_i$, 以及适当维数的矩阵 $X_{11i}, X_{12i}, X_{22i}, H_{1i}, H_{2i}$ 使得

$$(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1 < c_2 \lambda_1 e^{-\alpha T_f}.$$

其中 $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ 满足式(11), $\mu > 1$ 满足式(12)。

$$\begin{aligned} \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} &\geq 0, \\ \begin{bmatrix} X_{11i} & X_{12i} & H_{1i} \\ * & X_{22i} & H_{2i} \\ * & * & \tilde{T}_i \end{bmatrix} &\geq 0, \\ \begin{bmatrix} \varphi_{11} & \varphi_{12} & \tilde{P}_i C_i + A_i^T \tilde{Z}_i C_i + h_m A_i^T \tilde{T}_i C_i & \tilde{P}_i G_i + A_i^T \tilde{Z}_i G_i + h_m A_i^T \tilde{T}_i G_i \\ * & \varphi_{22} & B_i^T \tilde{Z}_i C_i + h_m B_i^T \tilde{T}_i C_i & B_i^T \tilde{Z}_i G_i + h_m B_i^T \tilde{T}_i G_i \\ * & * & \varphi_{33} & C_i^T \tilde{Z}_i G_i + h_m C_i^T \tilde{T}_i G_i \\ * & * & * & G_i^T \tilde{Z}_i G_i + h_m G_i^T \tilde{T}_i G_i - \frac{\tilde{M}_i}{r_m} \end{bmatrix} &< 0 \end{aligned}$$

其中,

$$\varphi_{11} = -\alpha \tilde{P}_i + \tilde{P}_i A_i + A_i^T \tilde{P}_i + \tilde{Q}_i + A_i^T \tilde{Z}_i A_i + h_m A_i^T \tilde{T}_i A_i + r_m \tilde{M}_i + H_{1i} + H_{1i}^T + h_m X_{11i},$$

$$\varphi_{12} = \tilde{P}_i B_i + A_i^T \tilde{Z}_i B_i + h_m A_i^T \tilde{T}_i B_i - H_{1i}^T + H_{2i} + h_m X_{12i},$$

$$\varphi_{22} = -(1 - \hat{h}) \tilde{Q}_i + B_i^T \tilde{Z}_i B_i + h_m B_i^T \tilde{T}_i B_i - H_{2i}^T - H_{2i} + h_m X_{22i},$$

$$\varphi_{33} = -(1 - \hat{\tau}) \tilde{Z}_i + C_i^T \tilde{Z}_i C_i + h_m C_i^T \tilde{T}_i C_i,$$

成立, 当平均驻留时间满足

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln[(\lambda_2 + h_m e^{\alpha h_m} \lambda_3 + \tau_m e^{\alpha \tau_m} \lambda_4 + h_m e^{\alpha h_m} \lambda_5 + r_m e^{\alpha r_m} \lambda_6) c_1] - \alpha T_f}$$

时, 切换系统(1)~(3)基于 $(c_1, c_2, T_f, \mathbf{R}, \sigma(t))$ 是有限时间稳定的。

证明 证明过程与定理1相似。证略。

2.2 有限时间增广的耗散性分析

定理2 考虑系统(1)~(3),令 $\tilde{\mathbf{P}}_i=\mathbf{R}^{\frac{1}{2}}\mathbf{P}_i\mathbf{R}^{\frac{1}{2}}$, $\tilde{\mathbf{Q}}_i=\mathbf{R}^{\frac{1}{2}}\mathbf{Q}_i\mathbf{R}^{\frac{1}{2}}$, $\tilde{\mathbf{Z}}_i=\mathbf{R}^{\frac{1}{2}}\mathbf{Z}_i\mathbf{R}^{\frac{1}{2}}$, $\tilde{\mathbf{T}}_i=\mathbf{R}^{\frac{1}{2}}\mathbf{T}_i\mathbf{R}^{\frac{1}{2}}$, $\tilde{\mathbf{M}}_i=\mathbf{R}^{\frac{1}{2}}\mathbf{M}_i\mathbf{R}^{\frac{1}{2}}$,对给定正的标量 $\alpha,h_m,\hat{h},\hat{\tau},r_m$,存在适当维数的正定对称矩阵 $\tilde{\mathbf{P}}_i,\tilde{\mathbf{Q}}_i,\tilde{\mathbf{Z}}_i,\tilde{\mathbf{T}}_i,\tilde{\mathbf{M}}_i$,以及适当维数的矩阵 $\mathbf{X}_{11i},\mathbf{X}_{12i},\mathbf{X}_{22i},\mathbf{H}_{1i},\mathbf{H}_{2i}$ 使得

$$\frac{1}{b}\tilde{\mathbf{P}}_i-\mathbf{F}_i^T\boldsymbol{\psi}_4\mathbf{F}_i>0, \quad (20)$$

$$\mathbf{X}_i=\begin{bmatrix} \mathbf{X}_{11i} & \mathbf{X}_{12i} \\ * & \mathbf{X}_{22i} \end{bmatrix} \geqslant 0, \quad (21)$$

$$\Delta_i=\begin{bmatrix} \mathbf{X}_{11i} & \mathbf{X}_{12i} & \mathbf{H}_{1i} \\ * & \mathbf{X}_{22i} & \mathbf{H}_{2i} \\ * & * & \tilde{\mathbf{T}}_i \end{bmatrix} \geqslant 0, \quad (22)$$

$$\Phi_i=\begin{bmatrix} \boldsymbol{\varphi}_{11} & \boldsymbol{\varphi}_{12} & \boldsymbol{\varphi}_{13} & \boldsymbol{\varphi}_{14} & \tilde{\mathbf{P}}_i\mathbf{G}_i+A_i^T\tilde{\mathbf{Z}}_i\mathbf{G}_i+h_m\mathbf{A}_i^T\tilde{\mathbf{T}}_i\mathbf{G}_i \\ * & \boldsymbol{\varphi}_{22} & \boldsymbol{\varphi}_{23} & \mathbf{B}_i^T\tilde{\mathbf{Z}}_i\mathbf{D}_i+h_m\mathbf{B}_i^T\tilde{\mathbf{T}}_i\mathbf{D}_i & \mathbf{B}_i^T\tilde{\mathbf{Z}}_i\mathbf{G}_i+h_m\mathbf{B}_i^T\tilde{\mathbf{T}}_i\mathbf{G}_i \\ * & * & \boldsymbol{\varphi}_{33} & \mathbf{C}_i^T\tilde{\mathbf{Z}}_i\mathbf{D}_i+h_m\mathbf{C}_i^T\tilde{\mathbf{T}}_i\mathbf{D}_i & \mathbf{C}_i^T\tilde{\mathbf{Z}}_i\mathbf{G}_i+h_m\mathbf{C}_i^T\tilde{\mathbf{T}}_i\mathbf{G}_i \\ * & * & * & \varphi_{44} & \mathbf{D}_i^T\tilde{\mathbf{Z}}_i\mathbf{G}_i+h_m\mathbf{D}_i^T\tilde{\mathbf{T}}_i\mathbf{G}_i \\ * & * & * & * & \mathbf{G}_i^T\tilde{\mathbf{Z}}_i\mathbf{G}_i+h_m\mathbf{G}_i^T\tilde{\mathbf{T}}_i\mathbf{G}_i-\frac{\tilde{\mathbf{M}}_i}{r_m} \end{bmatrix} < 0,$$

其中,

$$\begin{aligned} \boldsymbol{\varphi}_{11} &= -\alpha\tilde{\mathbf{P}}_i+\tilde{\mathbf{P}}_i\mathbf{A}_i+A_i^T\tilde{\mathbf{P}}_i+\tilde{\mathbf{Q}}_i+\mathbf{A}_i^T\tilde{\mathbf{Z}}_i\mathbf{A}_i+h_m\mathbf{A}_i^T\tilde{\mathbf{T}}_i\mathbf{A}_i-\mathbf{F}_i^T\boldsymbol{\psi}_1\mathbf{F}_i+r_m\tilde{\mathbf{M}}_i+\mathbf{H}_{1i}+\mathbf{H}_{1i}^T+h_m\mathbf{X}_{11i}, \\ \boldsymbol{\varphi}_{12} &= \tilde{\mathbf{P}}_i\mathbf{B}_i+A_i^T\tilde{\mathbf{Z}}_i\mathbf{B}_i+h_m\mathbf{A}_i^T\tilde{\mathbf{T}}_i\mathbf{B}_i-\mathbf{H}_{1i}^T+\mathbf{H}_{2i}+h_m\mathbf{X}_{12i}, \\ \boldsymbol{\varphi}_{13} &= \tilde{\mathbf{P}}_i\mathbf{C}_i+A_i^T\tilde{\mathbf{Z}}_i\mathbf{C}_i+h_m\mathbf{A}_i^T\tilde{\mathbf{T}}_i\mathbf{C}_i, \\ \boldsymbol{\varphi}_{14} &= \tilde{\mathbf{P}}_i\mathbf{D}_i+A_i^T\tilde{\mathbf{Z}}_i\mathbf{D}_i+h_m\mathbf{A}_i^T\tilde{\mathbf{T}}_i\mathbf{D}_i-\mathbf{F}_i^T\boldsymbol{\psi}_2, \\ \boldsymbol{\varphi}_{22} &= -(1-\hat{h})\tilde{\mathbf{Q}}_i+\mathbf{B}_i^T\tilde{\mathbf{Z}}_i\mathbf{B}_i+h_m\mathbf{B}_i^T\tilde{\mathbf{T}}_i\mathbf{B}_i-\mathbf{H}_{2i}^T-\mathbf{H}_{2i}+h_m\mathbf{X}_{22i}, \\ \boldsymbol{\varphi}_{23} &= \mathbf{B}_i^T\tilde{\mathbf{Z}}_i\mathbf{C}_i+h_m\mathbf{B}_i^T\tilde{\mathbf{T}}_i\mathbf{C}_i, \\ \boldsymbol{\varphi}_{33} &= -(1-\hat{\tau})\tilde{\mathbf{Z}}_i+\mathbf{C}_i^T\tilde{\mathbf{Z}}_i\mathbf{C}_i+h_m\mathbf{C}_i^T\tilde{\mathbf{T}}_i\mathbf{C}_i, \\ \boldsymbol{\varphi}_{44} &= -\boldsymbol{\psi}_3+\mathbf{D}_i^T\tilde{\mathbf{Z}}_i\mathbf{D}_i+h_m\mathbf{D}_i^T\tilde{\mathbf{T}}_i\mathbf{D}_i. \end{aligned} \quad (23)$$

$$\text{令 } \lambda_{\min}(\mathbf{P}_i)=\lambda_1, \lambda_{\max}(\mathbf{F}_i^T\mathbf{F}_i)=\lambda_8, \lambda_{\max}(\boldsymbol{\psi}_2^T\boldsymbol{\psi}_2)=\lambda_9, \lambda_{\max}(\boldsymbol{\psi}_3)=\lambda_{10}, \quad (24)$$

平均驻留时间满足

$$\tau_a > \tau_a^* = \frac{T_f \ln \mu}{\ln(\lambda_1 c_2) - \ln[\lambda_8 k + (\lambda_9 + \lambda_{10})d] - \alpha T_f}, \quad (25)$$

则系统满足增广的耗散性性能指标且基于 $(0,c_2,T_f,d,R,\sigma(t))$ 是有限时间有界的。

证明 与定理1的证明相似,可得:

$$\dot{V}(t)-\alpha V(t)-J(t)\leqslant \mathbf{X}^T(t)\boldsymbol{\Phi}_i\mathbf{X}(t)-\int_{t-h(t)}^t \boldsymbol{\vartheta}^T(t,s)\Delta_i\boldsymbol{\vartheta}(t,s)ds,$$

其中,

$$\begin{aligned} \mathbf{X}^T(t) &\stackrel{\Delta}{=} [\mathbf{x}^T(t)\mathbf{x}^T(t-h(t))\dot{\mathbf{x}}^T(t-\tau(t))\mathbf{w}^T(t)\int_{t-r(t)}^t \mathbf{x}^T(s) ds], \\ \boldsymbol{\vartheta}^T(t,s) &\stackrel{\Delta}{=} [\mathbf{x}^T(t)\mathbf{x}^T(t-h(t))\dot{\mathbf{x}}^T(s)], \end{aligned}$$

由式(22)~(23)得:

$$\dot{V}(t)-\alpha V(t)-J(t)<0,$$

与(14)的证明过程相似,从 0 到 t 积分得 $V(t) < e^{\alpha t} \mu^{N_\sigma(0,t)} V(0) + \int_0^t e^{\alpha(t-s)} \mu^{N_\sigma(s,t)} J(s) ds$,

在零初始状态 $V(0) = 0$ 下,得

$$V(t) < e^{\alpha t} \mu^{N_\sigma(0,t)} \int_0^t J(s) ds,$$

因此

$$\frac{V(t)}{e^{\alpha t} \mu^{N_\sigma(0,t)}} < \int_0^t J(s) ds,$$

由假设 4 得

$$\frac{V(t)}{b} < \int_0^t J(s) ds,$$

所以

$$\int_0^t J(s) ds \geq \frac{V(t)}{b} \geq \frac{1}{b} \mathbf{x}^T(t) \tilde{\mathbf{P}}_i \mathbf{x}(t) > 0.$$

考虑不等式

$$\int_0^{T_f} J(t) dt - \sup_{0 \leq t \leq T_f} \mathbf{z}^T(t) \psi_4 \mathbf{z}(t) \geq 0,$$

若 $\psi_4 = \mathbf{0}$, 显然 $\int_0^{T_f} J(t) dt \geq 0$ 。

若 $\psi_4 > \mathbf{0}$, 由假设 3 得 $\psi_1 = \mathbf{0}, \psi_2 = \mathbf{0}, \psi_3 > 0$, 所以

$$\int_0^t J(s) ds = \int_0^t \mathbf{w}^T(s) \psi_3 \mathbf{w}(s) ds,$$

因此对 $\forall t \in [0, T_f]$, $\int_0^{T_f} J(s) ds \geq \int_0^t J(s) ds \geq \frac{1}{b} \mathbf{x}^T(t) \tilde{\mathbf{P}}_i \mathbf{x}(t) > 0$, 由(20)有

$$\int_0^{T_f} J(s) ds \geq \frac{1}{b} \mathbf{x}^T(t) \tilde{\mathbf{P}}_i \mathbf{x}(t) \geq \mathbf{x}^T(t) \mathbf{F}_i^T \psi_4 \mathbf{F}_i \mathbf{x}(t) = \mathbf{z}^T(t) \psi_4 \mathbf{z}(t),$$

得

$$\int_0^{T_f} J(t) dt - \sup_{0 \leq t \leq T_f} \mathbf{z}^T(t) \psi_4 \mathbf{z}(t) \geq 0.$$

因此增广的耗散性证明完成,以下证明有限时间有界,由以上证明得

$$V(t) < e^{\alpha t} \mu^{N_\sigma(0,t)} \int_0^t J(s) ds,$$

从而

$$V(t) < e^{(\alpha + \frac{\ln n}{\tau_a}) T_f} \int_0^{T_f} J(s) ds,$$

又因为 $\psi_1 \leq \mathbf{0}$, 所以

$$\int_0^{T_f} J(s) ds \leq \int_0^{T_f} [2\mathbf{z}^T(s) \psi_2 \mathbf{w}(s) + \mathbf{w}^T(s) \psi_3 \mathbf{w}(s)] ds,$$

由以上不等式得

$$V(t) < e^{(\alpha + \frac{ln\mu}{\tau_a})T_f} \left[\int_0^{T_f} (2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)) ds \right],$$

则

$$x^T(t)R x(t) < \frac{V(t)}{\lambda_1} < \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} \left[\int_0^{T_f} [2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)] ds \right].$$

又因为

$$\int_0^{T_f} [2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)] ds = \int_0^{T_f} [2x^T(s)F_i^T \psi_2 w(s) + w^T(s)\psi_3 w(s)] ds.$$

由引理1得 $2x^T(s)F_i^T \psi_2 w(s) \leq x^T(s)F_i^T F_i x(s) + w^T(s)\psi_2^T \psi_2 w(s)$, 由假设2和式(24), 得

$$\begin{aligned} x^T(t)R x(t) &< \frac{V(t)}{\lambda_1} < \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} \left[\int_0^{T_f} [2z^T(s)\psi_2 w(s) + w^T(s)\psi_3 w(s)] ds \right] \\ &< \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} \left[\int_0^{T_f} (x^T(s)F_i^T F_i x(s) + w^T(s)\psi_2^T \psi_2 w(s) + w(s)\psi_3 w(s)) ds \right] \\ &< \frac{e^{(\alpha + \frac{ln\mu}{\tau_a})T_f}}{\lambda_1} [\lambda_8 k + (\lambda_9 + \lambda_{10})d]. \end{aligned}$$

则由式(25)得 $x^T(t)R x(t) < c_2$, 证毕。

3 仿真实例

考虑两个子系统的情况,令系统(1)~(2)的参数为

$$A_1 = \begin{bmatrix} 2 & 0 \\ 3 & 3 \end{bmatrix}, B_1 = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, C_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.2 \end{bmatrix}, D_1 = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.6 \end{bmatrix}, F_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.3 \end{bmatrix}, D_2 = \begin{bmatrix} -1 & 0 \\ 2 & 0.8 \end{bmatrix}, E_2 = \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.7 & 0 \\ 1 & 0.5 \end{bmatrix}, F_2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}.$$

对于有限时间有界分析,取 $\hat{h} = 0.01$, $\hat{\tau} = 0.01$, $r_m = 0.01$, $h_m = 0.01$, $\alpha = 0.01$, 解定理1中线性矩阵不等式(6)~(8)得,

$$\tilde{P}_1 = \begin{bmatrix} 18.0400 & -3.4414 \\ -3.4414 & 17.0201 \end{bmatrix}, \tilde{Q}_1 = \begin{bmatrix} 86.9766 & -0.0000 \\ -0.0000 & 86.9766 \end{bmatrix}, \tilde{Z}_1 = \begin{bmatrix} 11.9285 & -2.5917 \\ -2.5917 & 6.5873 \end{bmatrix},$$

$$\tilde{T}_1 = \begin{bmatrix} 74.5490 & -5.3528 \\ -5.3528 & 66.7197 \end{bmatrix}, \tilde{M}_1 = \begin{bmatrix} 1.5334 & 0.2300 \\ 0.2300 & 1.3243 \end{bmatrix}, N_1 = \begin{bmatrix} 149.9863 & 2.8596 \\ 2.8596 & 117.6619 \end{bmatrix},$$

$$X_{111} = \begin{bmatrix} 223.3248 & -148.3859 \\ -148.3859 & 217.1387 \end{bmatrix}, X_{121} = 10^3 \times \begin{bmatrix} 7.6390 & 1.7837 \\ 1.7837 & 7.4222 \end{bmatrix},$$

$$X_{221} = 10^3 \times \begin{bmatrix} 3.6669 & 1.0345 \\ 1.0345 & 3.6324 \end{bmatrix}, H_{11} = \begin{bmatrix} -194.5877 & -52.6727 \\ -52.6727 & -212.3848 \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} 54.7452 & 6.3773 \\ 6.3773 & 35.5525 \end{bmatrix},$$

$$\tilde{P}_2 = \begin{bmatrix} 0.7927 & 0.0895 \\ 0.0895 & 0.3913 \end{bmatrix}, \tilde{Q}_2 = \begin{bmatrix} 2.6209 & -0.0000 \\ -0.0000 & 2.6209 \end{bmatrix}, \tilde{Z}_2 = \begin{bmatrix} 0.7444 & 0.0948 \\ 0.0948 & 0.1792 \end{bmatrix},$$

$$\begin{aligned}\tilde{\mathbf{T}}_2 &= \begin{bmatrix} 2.459 & 1 & 0.108 & 9 \\ 0.108 & 9 & 2.000 & 0 \end{bmatrix}, \quad \tilde{\mathbf{M}}_2 = \begin{bmatrix} 0.048 & 6 & 0.002 & 3 \\ 0.002 & 3 & 0.033 & 3 \end{bmatrix}, \quad \mathbf{N}_2 = \begin{bmatrix} 4.757 & 7 & 0.407 & 6 \\ 0.407 & 6 & 2.842 & 1 \end{bmatrix}, \\ \mathbf{X}_{112} &= \begin{bmatrix} 20.834 & 4 & -0.591 & 9 \\ -0.591 & 9 & 1.910 & 6 \end{bmatrix}, \quad \mathbf{X}_{122} = \begin{bmatrix} 106.927 & 7 & 105.551 & 6 \\ 105.551 & 6 & -113.177 & 3 \end{bmatrix}, \\ \mathbf{X}_{222} &= \begin{bmatrix} 49.646 & 9 & 51.595 & 2 \\ 51.595 & 2 & -51.934 & 3 \end{bmatrix}, \quad \mathbf{H}_{12} = \begin{bmatrix} -6.999 & 9 & 2.134 & 9 \\ 2.134 & 9 & -5.765 & 8 \end{bmatrix}, \\ \mathbf{H}_{22} &= \begin{bmatrix} 1.434 & 1 & -0.093 & 1 \\ -0.093 & 1 & -0.019 & 3 \end{bmatrix}.\end{aligned}$$

对增广的耗散性性能指标,取 $\hat{h} = 0.01, \hat{\tau} = 0.01, r_m = 0.01, h_m = 0.01, \alpha = 0.01$, 对应评论 1 中选取不同的矩阵,以 H_∞ 性能指标为例,取 $\psi_1 = -I, \psi_2 = \mathbf{0}, \psi_3 = \gamma^2 I, \psi_4 = \mathbf{0}$, 解定理 2 中的线性矩阵不等式(21)–(23)得最优值 $\gamma_{\min}^2 = 1 * 10^{-11}$, 且

$$\begin{aligned}\tilde{\mathbf{P}}_1 &= \begin{bmatrix} 0.013 & 3 & -0.000 & 2 \\ -0.000 & 2 & 0.009 & 9 \end{bmatrix}, \quad \tilde{\mathbf{Q}}_1 = 10^8 \times \begin{bmatrix} 2.907 & 7 & 0.025 & 1 \\ 0.025 & 1 & 2.942 & 3 \end{bmatrix}, \\ \tilde{\mathbf{Z}}_1 &= 10^{-11} \times \begin{bmatrix} 0.277 & 4 & -0.055 & 9 \\ -0.055 & 9 & 0.112 & 8 \end{bmatrix} \\ \tilde{\mathbf{T}}_1 &= 10^{-9} \times \begin{bmatrix} 0.100 & 1 & -0.025 & 8 \\ -0.025 & 8 & 0.042 & 6 \end{bmatrix}, \quad \tilde{\mathbf{M}}_1 = 10^7 \times \begin{bmatrix} 5.646 & 3 & 0.087 & 9 \\ 0.087 & 9 & 5.535 & 3 \end{bmatrix} \\ \mathbf{X}_{111} &= 10^6 \times \begin{bmatrix} -2.557 & 0 & 0.091 & 7 \\ 0.091 & 7 & -2.517 & 1 \end{bmatrix}, \quad \mathbf{X}_{121} = 10^6 \times \begin{bmatrix} -1.543 & 4 & -0.072 & 7 \\ -0.072 & 7 & -1.646 & 1 \end{bmatrix}, \\ \mathbf{X}_{221} &= 10^6 \times \begin{bmatrix} -1.666 & 4 & -0.027 & 8 \\ -0.027 & 8 & -1.714 & 6 \end{bmatrix}, \quad \mathbf{H}_{11} = 10^8 \times \begin{bmatrix} -5.113 & 9 & 0.183 & 3 \\ 0.183 & 3 & -5.034 & 1 \end{bmatrix}, \\ \mathbf{H}_{21} &= 10^8 \times \begin{bmatrix} 1.789 & 4 & -0.017 & 1 \\ -0.017 & 1 & 1.783 & 0 \end{bmatrix}, \\ \tilde{\mathbf{P}}_2 &= \begin{bmatrix} 0.025 & 5 & 0.005 & 8 \\ 0.005 & 8 & 0.010 & 0 \end{bmatrix}, \quad \tilde{\mathbf{Q}}_2 = 10^7 \times \begin{bmatrix} 3.498 & 2 & 0.000 & 3 \\ 0.000 & 3 & 3.495 & 2 \end{bmatrix}, \quad \tilde{\mathbf{Z}}_2 = 10^{-9} \times \begin{bmatrix} 0.124 & 6 & 0.041 & 1 \\ 0.041 & 1 & 0.024 & 6 \end{bmatrix}, \\ \tilde{\mathbf{T}}_2 &= 10^{-8} \times \begin{bmatrix} 0.570 & 0 & 0.213 & 4 \\ 0.213 & 4 & 0.109 & 4 \end{bmatrix}, \quad \tilde{\mathbf{M}}_2 = 10^5 \times \begin{bmatrix} 6.947 & 3 & -0.003 & 8 \\ -0.003 & 8 & 6.964 & 5 \end{bmatrix}, \\ \mathbf{X}_{112} &= 10^5 \times \begin{bmatrix} -2.352 & 3 & -0.083 & 3 \\ -0.083 & 3 & -2.187 & 5 \end{bmatrix}, \quad \mathbf{X}_{122} = 10^6 \times \begin{bmatrix} -2.095 & 4 & -0.953 & 2 \\ -0.953 & 2 & -1.177 & 0 \end{bmatrix}, \\ \mathbf{X}_{222} &= 10^6 \times \begin{bmatrix} -1.048 & 5 & -0.476 & 5 \\ -0.476 & 5 & -0.589 & 4 \end{bmatrix}, \quad \mathbf{H}_{12} = 10^7 \times \begin{bmatrix} -4.704 & 6 & -0.166 & 7 \\ -0.166 & 7 & -4.374 & 9 \end{bmatrix}, \\ \mathbf{H}_{22} &= 10^5 \times \begin{bmatrix} 1.603 & 1 & -0.198 & 6 \\ -0.198 & 6 & 1.790 & 0 \end{bmatrix}.\end{aligned}$$

以上仿真结果表明,存在相应的矩阵使得相应的矩阵不等式成立,系统有限时间有界与增广的耗散性成立,从而验证了方法的有效性。

4 结论

运用平均驻留时间及线性矩阵不等式方法分析了系统有限时间有界与增广的耗散性问题,基于增广的耗散性概念,将 $H_\infty, L_2 - L_\infty$, 无源性, (Q, S, R) –耗散性指标整合到一个统一的框架,使得系统性能的分析更加方便。用线性矩阵不等式方法可使仿真结果验证更加便利。未来增广的耗散性概念将会应用到更多复杂动态系统中。

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